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LASER DOPPLER SHIFT VELOCIMETER

by

F. N. Weber, Jr.

April 1972

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APRIL 1972

LASER DOPPLER SHIFT VELOCIMETER

F. N. Weber, Jr.

Exterior Ballistics Laboratory

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Finally some results of a literature study concerning the use of laser doppler velocimetry as applied to the determination of the speed of wind tunnel flows are given.

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FNWeber, Jr./so
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Two different schemes for the laser doppler velocimeter are described and theoretically analyzed, a modified Mach-Zehnder velocimeter, and a modified Rayleigh velocimeter. Preliminary results obtained for the first scheme are shown and rather extensive results for the second scheme are presented. This latter method is shown to be capable under rather controlled conditions of obtaining speeds to well within 1%.

Finally some results of a literature study concerning the use of laser doppler velocimetry as applied to the determination of the speed of wind tunnel flows are given.

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I. INTRODUCTION

Theory: It is well known that an observer moving with respect to a source will hear a frequency ν_o which is different from that of the source of the sound, ν_s. This Doppler effect occurs for electromagnetic radiation as well as sound. Because in the literature many authors, in an attempt to simplify the derivation for electromagnetic radiation as applied to velocimetry, have confused it to the point of making it unintelligible, a full derivation will be given here (See Figure 1.)

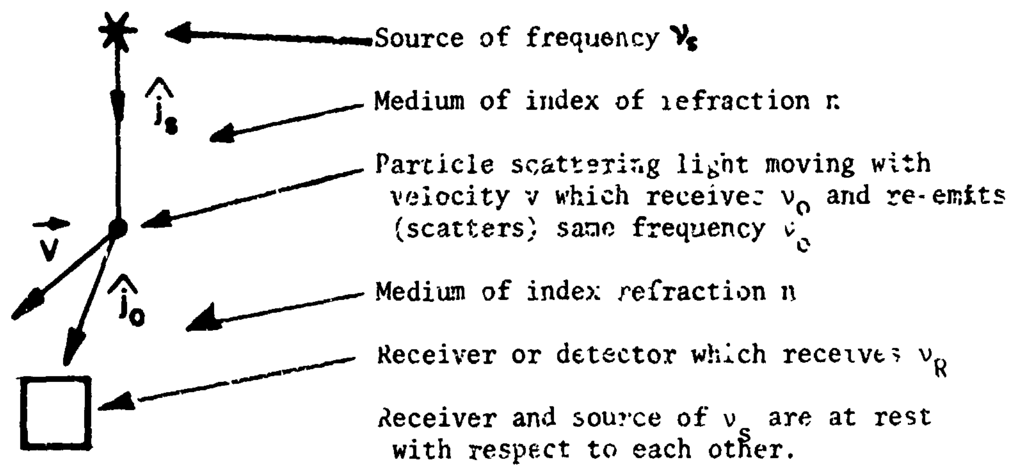


Figure 1. Geometry of Scattering Process

We now derive the expression for ν_o, the frequency of radiation received by the moving observer. The Lorentz time relation, between system S (x, y, z, t) and S' (x', y', z', t') moving with a constant velocity v with respect to S along the x axis is given by[1]*

$$t' = \frac{t_1 - (v/c^2)\, x_1}{\sqrt{1-v^2/c^2}} \qquad (1)$$

*References are listed on page 25.

where x is the displacement of the event measured in S, t is time at which the event was at x, and t' is time measured in S'.

Consider the sequence of events that is depicted in Figure 2. Unprimed variables refer to the coordinate system of the radiation source, i.e., the system with respect to which the radiation source is at rest.

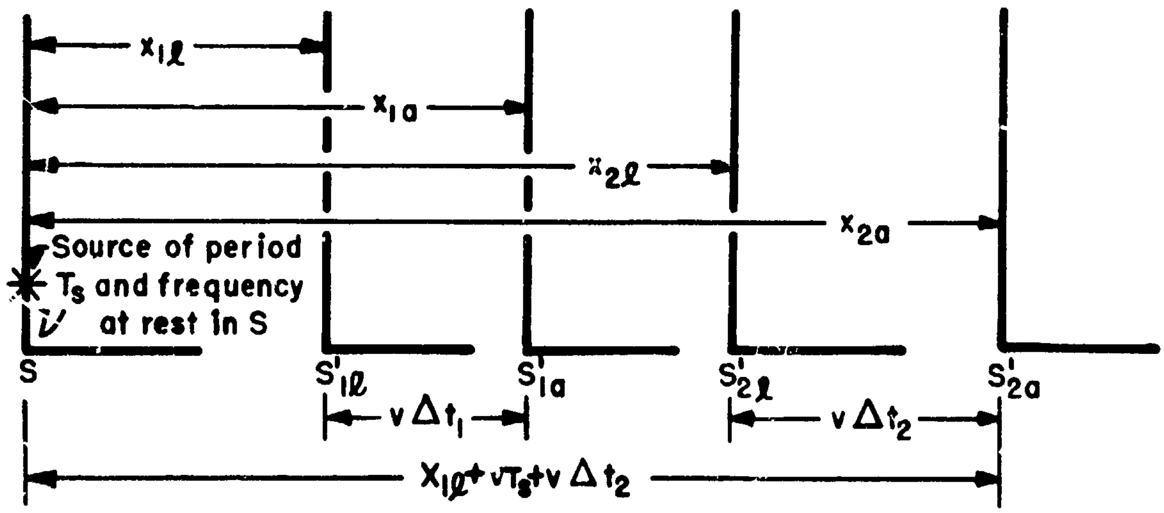


Figure 2. Systems to Compute Doppler Shift

Primed variables refer to the coordinate system of the observer. Numerical subscripts refer to the number of oscillations being considered, whereas the subscript letter $\underline{\ell}$ indicates the oscillation is leaving the source and $\underline{a}$ indicates the oscillation is arriving at the observer. Hence, for example $x_{1\ell}$ is the x coordinate, measured by the observer fixed with respect to the source, of the location of the moving observer at the instant the 1st oscillation leaves the source. t'_{1a} is the arrival time of this oscillation as measured by the moving observer.

Finally it should be evident that the period of this radiation as determined by the unprimed observer is $t_{2\ell} - t_{1\ell}$, whereas that determined by the primed (moving) observer is $t_{2a} - t_{1a}$. These we designate at T_s and T_o respectively.

Hence from Eq. (1)

$$t'_{2a} - t'_{1a} \equiv T_o = \frac{t_{2a} - t_{1a} - \frac{v}{c^2}(x_{2a} - x_{1a})}{\sqrt{1 - v^2/c^2}} . \tag{2}$$

Now $t_{2a} = t_{2\ell} + \Delta t_2$ and $t_{1a} = t_{1\ell} + \Delta t_1$, where Δt_2 and Δt_1 are the transit times for the radiation between the coordinate systems for the second and first pulse respectively as measured by the fixed observer. Hence

$$\Delta t_2 = \frac{x_{1\ell} + vT_s + v\Delta t_2}{v_w} \quad \text{and}$$
$$\Delta t_1 = \frac{x_{1\ell} + v\Delta t_1}{v_w} \tag{3}$$

where v_w is the speed of the radiation in the medium. Note also

$$t_{1a} = t_{1\ell} + \Delta t_1 \text{ and } t_{2a} = t_{2\ell} + \Delta t_2 \quad . \tag{4}$$

Finally it is evident from Figure 2 that

$$x_{1a} = x_{1\ell} + v\Delta t_1 \text{ and } x_{2a} = x_{1\ell} + vT_s + v\Delta t_2 \; . \tag{5}$$

If Eqs. (3) are substituted into Eqs. (4) and (5), and the resulting equations are substituted into Eq. (2) one gets

$$T_o = \sqrt{1 - v^2/c^2} \left(\frac{v_w}{v_w - v}\right) T_s \; .$$

Since we are more interested in frequencies,

$$\nu_o = \frac{\nu_s}{\sqrt{1-v^2/c^2}}(1 - \frac{v}{v_w}) .$$

This is the Doppler relationship for the case where the receiver is moving parallel to the direction of propagation of the radiation. Recall v is the receiver speed in the direction of propagation and v_w is the speed of the radiation with respect to the source of frequency ν_s. Where the receiver is moving toward the source, the v changes sign. Hence

$$\nu_o = \nu_s \frac{(1 \mp v/v_w)}{\sqrt{1 - v^2/c^2}} . \tag{6}$$

The upper sign applies to a receding source, the lower to an approaching source. If we define $\hat{j}$ as a unit vector in the direction of propagation of the radiation and let $\vec{v}$ be the velocity vector describing the motion of the receiver relative to the source, we may easily generalize Eq. (6) into

$$\nu_o = \nu_s (1-\hat{j}\cdot\frac{\vec{v}}{v_w})/(1-v^2/c^2)^{1/2} . \tag{7}$$

Note that even when the receiver is moving perpendicularly to the direction of propagation of the light ($\hat{j}\cdot\vec{v} = 0$), there is a Doppler shift called the second order or transverse Doppler shift. Also note that Eq. (7) applies to any periodic disturbance. For example, for a sound source approaching an observer who is at rest in the medium where the speed of sound is s, Eq. (7) becomes

$$\nu_o = \nu_s \frac{(1 + \frac{v}{s-v})}{\sqrt{1-v^2/c^2}}$$

$$\cong \frac{\nu_s s}{s-v}$$

which is the standard result for sound when the observer is at rest in the medium and the source is moving toward him at speed v. One can proceed for the other three cases of the Doppler effect for sound, and Eq. (7) will give the correct value.

For electromagnetic radiation in a medium of index $n = 1$, $v_w \equiv c$, and Eq. (7) gives the familiar expression

$$\nu_o = \nu_s \frac{(1 - \frac{\vec{v}}{c} \cdot \hat{j}_s)}{(1 - v^2/c^2)^{1/2}}$$

where $\hat{j}_s$ is the unit vector in the direction of propagation of the source. In the case of Doppler shifting with light in a medium of index of refraction $n \neq 1$, $v_w = \frac{c}{n}$. Hence Eq. (7) becomes

$$\nu_o = \nu_s \frac{(1 - n \frac{\vec{v}}{c} \cdot \hat{j}_s)}{(1 - v^2/c^2)^{1/2}} . \tag{8}$$

Refer to the geometry of Figure 1. In general the frequency that the receiver in Figure 1 receives will be different than ν_o by still another Doppler shifting and in the geometry of Figure 1, the received frequency ν_r is given from Eq. (7) as

$$\nu_r = \nu_o \frac{(1 + n \frac{\vec{v}}{c} \cdot \hat{j}_o)}{(1 - v^2/c^2)^{1/2}} .$$

Including ν_o from Eq. (8),

$$\nu_r = \nu_s \frac{(1 - n \frac{\vec{v}}{c} \cdot \hat{j}_s)(1 + n \frac{\vec{v}}{c} \cdot \hat{j}_o)}{(1 - v^2/c^2)} . \tag{9}$$

Eq. (9) is the general equation for the frequency of the radiation received from a scatterer, moving at velocity $\vec{v}$.

In most applications terms of v^2/c^2 may be dropped, and Eq. (9) becomes

$$\nu_r = \nu_s \left[1 - n \frac{\vec{v}}{c} \cdot (\hat{j}_s - \hat{j}_o)\right] . \qquad (10)$$

This theory will now be applied to two general methods of heterodyning.

II. METHOD I

From Figure 3, familiar to those using Doppler shifting to measure v, $\vec{v} \cdot \hat{j}_s = v \cos \theta$ and $\vec{v} \cdot \hat{j}_o = 0$.

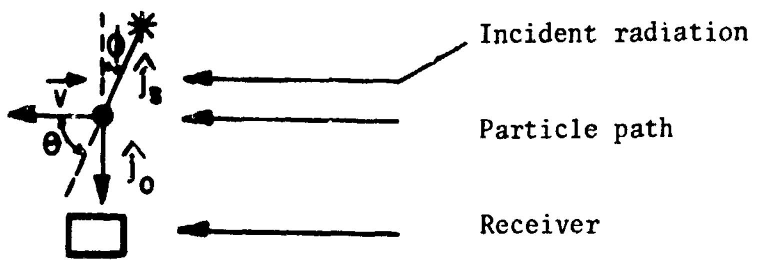


Figure 3. Fundamentals of Geometry for Method I.

Hence $\nu_r = \nu_s \left(1 - n \frac{v \sin \phi}{c}\right)$ where ϕ is angle of incidence as shown in Figure 3.

If the two frequencies, ν_r and ν_s are beat or heterodyned together,

$$\nu_H \equiv |\nu_r - \nu_s| = \nu_s n \frac{v \sin \phi}{c} . \qquad (11)$$

Hence since $\nu_s/c = 1/\lambda_o$, where the vacuum wavelength of the radiation is designated by λ_o, Eq. (11) becomes

$$\nu_H = n \frac{v \sin \phi}{\lambda_o}$$

The frequency ν_H is referred to as the heterodyne frequency. A typical use of this method is seen in Figure 4.

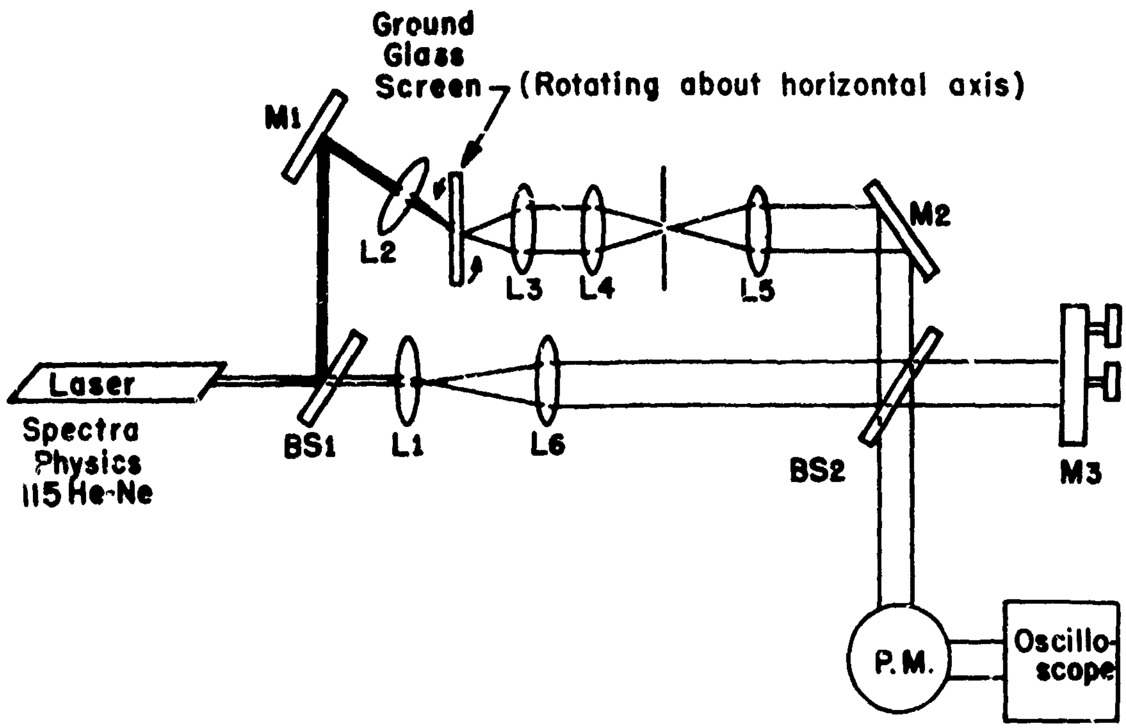


Figure 4. Method I. Modified Mach-Zehnder

Alignment and Adjustment of Optical System for Method I.

The optical system must be very precisely adjusted or heterodyning will not occur The most critical alignment is between the light coming off M2 and that of M3. These two beams should be coaxial to the highest degree possible. The procedure for their alignment is to place a ground glass screen close to BS2 and by adjusting M3 and BS2 have the two spots, caused by the two beams, coincide. Then move toward the photomultiplier, P.M, approximately 5 feet and check to make sure the two spots continue to coincide. In general they will not, and several trials of adjusting BS2 and M3 must be made until the spots continue to coincide over the 5 foot interval. When this has been done, the output of the RCA 6217 photomultiplier is fed into a Krohn-Hite Model 3100R Electronic Filter and then into a Tektronix 551 oscilloscope on the 5 millivolt/cm scale and final fine adjustment is made by turning the

adjusting screws on M3. When optical heterodyning occurs, a noisy pattern will suddenly spring into a smooth sinusoid of amplitude very much greater than the noise observed just prior to heterodyning. Many optical components are required in this method in order to have the two separate beams alike upon their recombination.

III. METHOD II

Figure 5 shows the fundamentals of the second method of optical heterodyning tested. The two beams heterodyne at their intersection point, the heterodyning occuring because at their intersection the scattering particle is approaching the source of one beam and receding from the other.

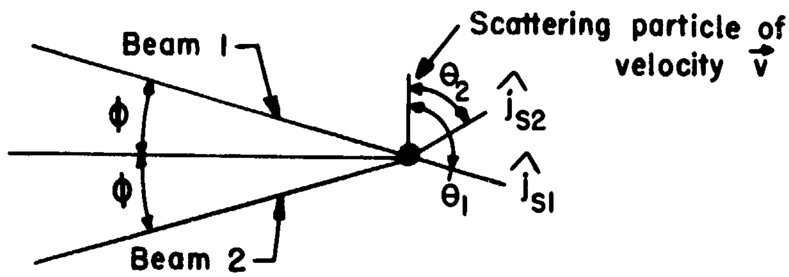


Figure 5. Fundamentals of Geometry for Method II

From Eq. (8) $\nu_{01} \equiv$ the frequency observed by the scattering particle due to beam 1.

$$\nu_{01} = \nu_s \left(1 - \frac{nv}{c} \cos \theta_1\right)$$

$$= \nu_s \left(1 + \frac{nv}{c} \sin \phi\right).$$

For Beam 2

$$\nu_{02} = \nu_s \left(1 - \frac{nv}{c} \cos \theta_2\right)$$

$$= \nu_s \left(1 - \frac{nv}{c} \sin \phi\right).$$

$$\nu_H \equiv |\nu_{01} - \nu_{02}|$$

$$= \frac{2n\nu\nu_s \sin\phi}{c} \; . \qquad (13)$$

The setup of this type tested in the laboratory is seen in Figure 6.

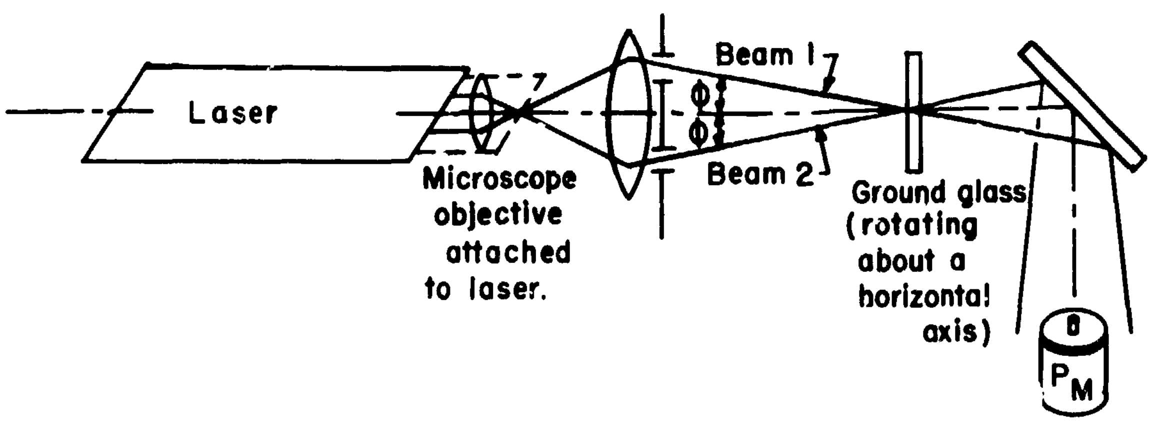


Figure 6. Method II - Modified Rayleigh

<u>Alignment and Adjustment of Optical System for Method II.</u>

This optical system, relative to Method I, is exceedingly trivial to align. Given the proper components an experienced operator can align the system in a matter of a few minutes. The same person might spend several hours aligning system I.

IV. ANALYSIS OF HETERODYNE SIGNAL

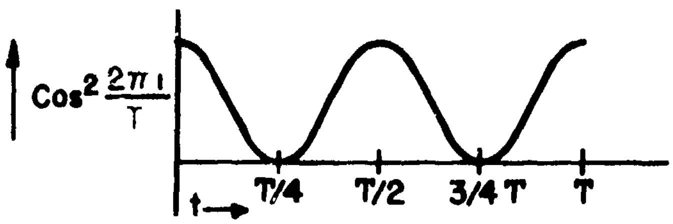
One can show* that the patterns seen on the oscilloscope screen

See Appendix page 23

are given by the expression

$$I = I_o \cos^2 2\pi \left(\frac{t}{2T_H}\right) .$$

A cosine squared function $\cos^2 2\pi t$ over one period is represented below.



Thus if one determines the period T of the cosine squared pattern from the oscilloscope, $T_H = \frac{T}{2}$.

V. TRIAL MEASUREMENTS

The theory and experimental set-up for these two methods were checked in measurements with the rotating ground glass screen. For Method I the motor rotating the screen was a clock motor that turned at 1.00 rev/sec. Thus the speed v was

$$v = 2\pi\ r\ (cm)\ cm/sec \qquad (14)$$

where r is measured in units of centimeters. Two r's were used at the same angle ϕ (see Eq. (12)), r = 2 and 2 = 4. Comparison of ν_H's for each showed that ν_{H_2} for the 2 cm radius = $\frac{0.95}{1.87}$ ν_{H_4}, where ν_{H_4} is the Doppler frequency for the 4 cm radius. Of course the theory predicts that $\nu_{H_2} = 0.500\ \nu_{H_4}$, but the difference of slightly more than 1.5% is not regarded as significant since it was difficult to measure r to better than a couple of percent.

Another check was made of the accuracy of this set-up. This was an absolute determination of v using Eq. (12). The value of v from Eq. (12) was compared with the "theoretical" value from Eq. (14). For

$r = 2$ cm, $\phi \cong (8 \pm .5^o)$, T_H from the oscilloscope photograph $= 1.87$ cm x 20 μsec/cm $= 3.74 \times 10^{-5}$ sec. $\therefore$ $\nu_{H_2} \approx [3.74 \times 10^{-5} \text{ sec}]^{-1}$ $= 26.7$ KH hence from Eq. (12)

$$v = \frac{26.7 \times 10^3 \times 6.33 \times 10^{-5}}{1 \times \sin 8^o} \text{ cm/sec} \equiv v_{EX} .$$

Hence

$$v_{EX} = 12.15 \text{ cm/sec, whereas}$$

the "theoretical" value of $v = 2\pi \times 2$ cm/sec

$$v_{TH} = 12.57 \text{ cm/sec.}$$

The difference between v_{EX} and v_{TH} can easily be accounted for by the lack of precision in measuring ϕ.

For Method II many more careful and extensive tests were made. First of all it is evident from the geometry of Figure 6 that the angle ϕ can be determined simply and quite accurately (well within 1%) from simple measurements. In testing Method II a new motor and wheel were used. This system rotated at approximately 1000 rpm or 600 rpm with a radius of approximately 4 in. Table I shows some of the results obtained with different angles (ϕ's) and speeds.

It can be seen that for the first seven values there is large random error in v_{EX}; the last six values are within one percent and consistently slightly high. The explanation for the random high errors can be seen from the following analysis.

Recall from Eq. (13)

$$\nu_H = 2n \frac{v\nu_s \sin \phi}{c}$$

$$= \frac{2n\ v \sin \phi}{\lambda_o}$$

Table I

Angle x 10^{+3} (radians)	ν_H x 10^{-5} (H_z)	v_{EX} x 10^{-2} (cm/sec)	v_{TH} x 10^{-2} (cm/sec)	Difference + $\Rightarrow v_{EX} > v_{TH}$
11.99	4.30	11.3	11.0	+ 2.8%
36.06	12.90	11.33	11.35	- .1%
4.776	1.702	10.1	10.6	- 5%
4.776	1.685	10.0	10.4	- 4%
5.218	1.736	10.5	10.4	+ 1%
5.218		10.4	10.4	- .1%
5.218		10.6	10.2	+ 4%
37.74	6.579	5.99	5.96	+ .5%
34.74	6.579	5.99	5.96	+ .5%
34.74	6.547	5.96	5.91	+ .85%
34.74	6.544	5.96	5.92	+ .68%
34.74	6.484	5.91	5.86	+ .85%
34.74	6.436	5.86	5.83	+ .51%

where λ_o is the vacuum wavelength of the laser (6328 Å).
Hence

$$\delta\nu_H = \frac{2nv}{\lambda_o} \cos\phi\ \delta\phi$$

considering only variations in the angle ϕ. Hence

$$\frac{\delta\nu_H}{\nu_H} = \frac{\cos\phi\ \delta\phi}{\sin\phi} = \cot\phi\ \delta\phi.$$

Hence $\delta\nu_H/\nu_H$ which is often referred to in the literature as the instrumental band width is a function of the slit separation w and their width δw. Since the separation w is a determining factor in the angle ϕ above, $\phi = \tan^{-1} w/L$ where L is the distance from the converging lens to the point of intersection of the two beams, then by increasing w one may

decrease cot ϕ and the instrumental band width. Likewise by decreasing w, i.e., by making δw smaller, one may likewise decrease the band width. Both of these measures were taken in the latter readings of Table I. The angle is larger than most of the preceeding values by a factor of about seven and δw is smaller by about a factor of 2. ($\delta \approx .010$ in). A comparison of two typical photographs seen in Figures 7 and 8 shows the difference between the two patterns. Figure 7 was taken when the band width was large. Figure 8 was taken when the steps outlined above had been taken to make $\delta\nu_H/\nu_H$ as small as practicable. If Figure 7 is examined closely it can be seen that there is both a frequency and amplitude modulation present. The reason for the amplitude modulation is probably variations in the intensity of the light transmitted through the ground glass screen. The frequency modulation is a direct consequence of a large instrumental band width, eliminated in Figure 8. Thus if steps are taken to keep $\delta\nu_H/\nu_H$ small, an accurate (within 1%) determination of the linear speed of the rotating ground glass screen can be made.

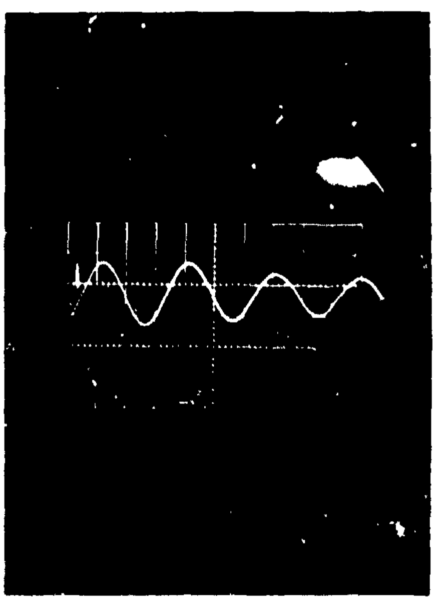


Figure 7. Photomultiplier Output With Large Band Width

Figure 8. Photomultiplier Output Slit Width, δw Decreased.

VI. SUITABILITY FOR WIND TUNNEL USE

It has been the hope of many wind tunnel personnel that the laser doppler velocimeter could be used for determining the velocity of the gas flow in their facilities. These workers point out the fact that the laser velocimeter can determine this velocity without the introduction of probes that would disturb the flow. They also point out that the measurements would not depend on a combination of indirect thermodynamic measurements but would be direct. Indeed these advantages have given the initial impetus to this work. However, the work presented here can hardly be extrapolated to an almost entirely different situtation: the velocity of gas particles. What has been shown in the above work is that the laser velocimeter can determine with about 1% accuracy, the speed of large (with respect to the wavelength of light of the laser) particles moving at constant velocities. In the wind tunnel one would be using the laser velocimeter to determine the velocity of small (with respect to the laser wavelength) particles which do not in general move with a constant velocity.

The work done by Yanta et al[2], indicates the extent of these problems in use of the laser velocimeter in the wind tunnel. First of all since the gases in the tunnels scatter indetectably small amounts of light, the flows must be "seeded" with large particles which scatter significant quantities of light. The seeding problem is an exceedingly difficult one: if large particles are used in accelerating flows, then the velocities determined will be those of the large particles which can lag seriously the gas flows[2]. If smaller particles are used, insufficient quantities of light are scattered and the signal to noise ratio is very poor[3]. In either case it is extremely difficult to generate uniformly sized seeding particles and if the extrapolation backward to smaller particle behavior is attempted (as suggested by Yanta, et al[2]), the results would be most tenuous. This problem of extrapolation is further very much complicated by the fact that the particles in the size range used for seeding (.1 to 10 μ) scatter light the quantity of which depend upon their diameter. One may show using the methods of

Mie as applied to the scattering of light by particles comparable in size to the incident radiation wavelength that for example a 1μ particle scatters approximately 10^4 times the light of a .1μ particle! (Note Ref. 2 is in error here; it claims a 10^2 difference.) Thus, if a small amount of impurity with respect to size homogeneity occurs, one is likely to be determining the flow velocity of the large sized impurities while assuming that he is finding the velocities of the smaller particles. When the extrapolations are done back to the gas particles, the results are in large error. Ref. 2 suggests that size variations of a factor of 10 are not uncommon.

The velocimeter of Method II tested in this laboratory improves on the accuracy of those reported in the literature (typically 2 to 3% error) by a couple of percent, for the determination of the linear speed of a ground glass screen. The sampling distance is also smaller than others reported (approximately .1 in. for the velocimeter described as Method II versus for example that of Ref. 2 which has a sampling length of 2 in.!) However these improvements do not eliminate the serious problems associated with seeding mentioned above. Unless a method of seeding is developed (it is not yet, according to the open literature) that injects highly homogeneous particles into the flow, use of a laser velocimeter to find velocities of accelerating flows (most flows of interest) seems fraught with difficulties and inaccuracies. If however, the flows are of low or zero acceleration the laser velocimeter can be an accurate tool for determining the velocities of these flows.

APPENDIX

Calculation of Doppler Frequency Equation

We begin by assuming that the beams are coaxial and parallel. Thus they are in a fixed phase across their cross-section and their electric vectors are parallel across their cross-section. Hence when we compute the resulting magnitude of the electric field of their combination, we need merely add algebraically to obtain their resultant magnitude.

Let the electric vector of one beam be $\vec{E}_1$ and that of the other $\vec{E}_2$. Let their combination be $\vec{E}$.

Hence $\vec{E} = \vec{E}_1 + \vec{E}_2$, but since parallel, we can write $|\vec{E}| = |\vec{E}_1|+|\vec{E}_2|$. We will write E for $|\vec{E}|$, etc., so our equation becomes merely $E = E_1 + E_2$. Assume $E_1 = E_{1_0} \sin 2\pi\nu_1 t$ and $E_2 = E_{1_0} \sin (2\pi\nu_2 t + \alpha)$. Note here the ν's are different (due to the Doppler effect), the amplitudes of the waves have been assumed to be the same (easily achieved with neutral density filters) and the two beams are α out of phase.

Hence,
$$\frac{E}{E_{1_0}} = \sin 2\pi\nu_1 t + \sin (2\pi\nu_2 t + \alpha) \qquad (1A)$$

Using the trigonometric identity for the sum of the sines of two angles $\sin A + \sin B = 2 \sin \left(\frac{A+B}{2}\right) \cos \left(\frac{A-B}{2}\right)$ we may write Eq. (1A) as

$$\frac{E}{2E_{1_0}} = \sin\left(\frac{2\pi\nu_1 t + 2\pi\nu_2 t + \alpha}{2}\right) \cos\left(\frac{2\pi\nu_1 t - 2\pi\nu_2 t - \alpha}{2}\right)$$

We recognize that $\frac{\nu_1 + \nu_2}{2}$ is the average of the two frequencies and designate it $\bar{\nu}$. Hence we have

$$\frac{E}{2E_{1_0}} = \sin\left(2\pi\bar{\nu}t + \frac{\alpha}{2}\right)\left\{\cos\left[2\pi\left(\frac{\nu_1 - \nu_2}{2}\right)t - \frac{\alpha}{2}\right]\right\}$$

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We can think of the curly brackets as an amplitude of the high frequency $\sin\left(2\pi\bar{\nu}t + \frac{\alpha}{2}\right)$ term.

Since we are interested in the intensity I' of light (the P.M. is sensitive to light intensity, not E), we need to look at E^2 ($I' = E^2$ x a constant $= kE^2$).

Let $\sin\left(2\pi\bar{\nu}t + \frac{\alpha}{2}\right) \equiv f(t)$, then

$$I' = k^2 \cos^2\left[2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t - \frac{\alpha}{2}\right]f(t)^2$$

This is what the photomultiplier "sees". However, since it can never respond to the rapidly varying $[f(t)]^2$ term, the signal input to the oscilloscope is merely $k^2 \cos^2\left[2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t - \frac{\alpha}{2}\right]$.

Realizing k^2 is an amplitude (we set it $= I_o$) and the phase factor $\frac{\alpha}{2}$ doesn't change the wave form, we can write $I = I_o \cos^2\left[2\pi\left(\frac{\nu_1-\nu_2}{2}\right)t\right]$ or since $\nu_1 - \nu_2 = \nu_H$, $I = I_o \cos^2\left[2\pi\left(\frac{t}{2T_H}\right)\right]$ which is the equation found in the text.

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